Succinct Representations for Abstract Interpretation

Combined Analysis Algorithms and Experimental Evaluation.

Julien Henry, David Monniaux, Matthieu Moy

September 13, 2012

Sources of Imprecision in Abstract Interpretation

- Abstract domain
- Widening operator
  - ensures fast convergence
  - BUT: may induce huge imprecisions
  - Narrowing tends to recover some precision…
- Consider paths that are unfeasible in reality: least upper bound operations

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Summary

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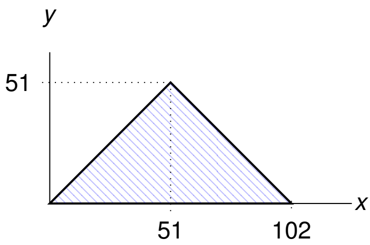
1. Introduction: Weakness of the standard approach & Guided Static Analysis
2. Using SMT-solving to focus new paths
3. Combining Both Techniques
4. Computing Disjunctive Invariants

Example of Standard Abstract Interpretation

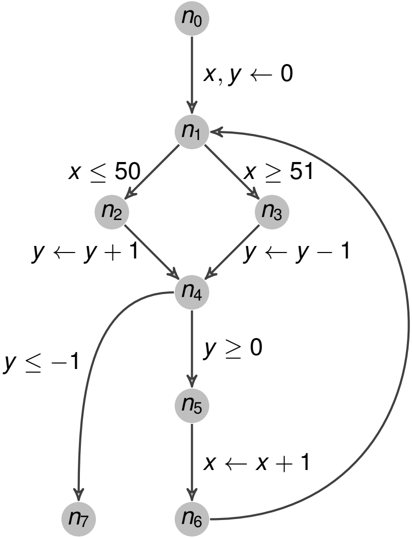
Example from Gopan & Reps, SAS'07

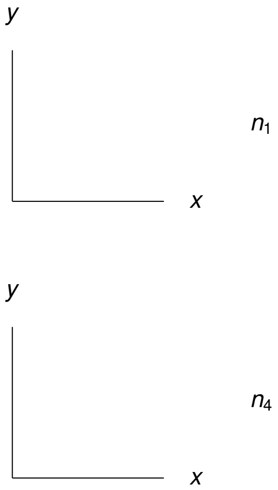
```
x = 0;
y = 0;
while (true) {
        if (x <= 50)
                y++;
        else
                y--;

        if (y < 0) break;
        x++;
}
```



- x and y incremented during 51 iterations
- x incremented and y decremented during 51 iterations

Example of Standard Abstract Interpretation





Ascending iterations

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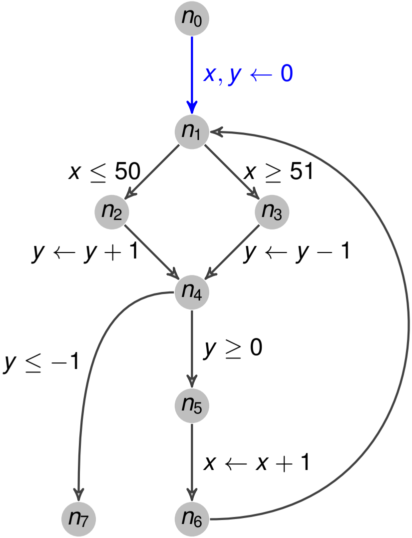

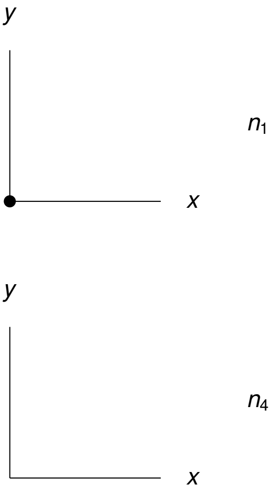

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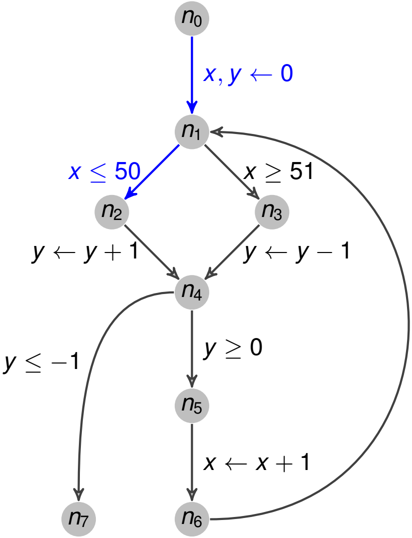

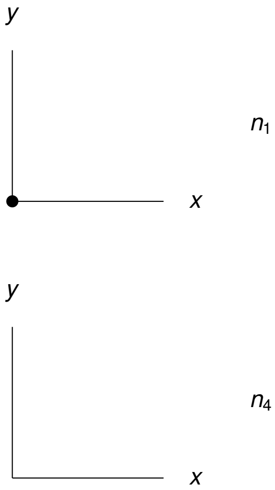

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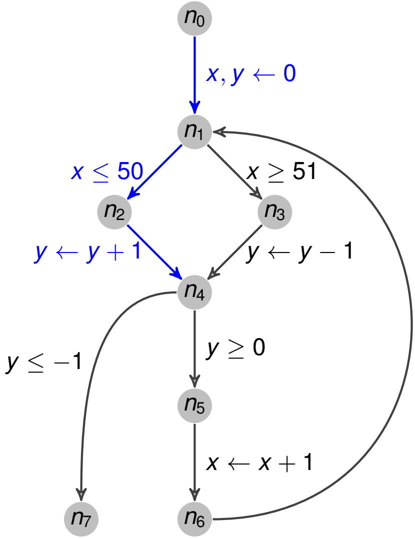

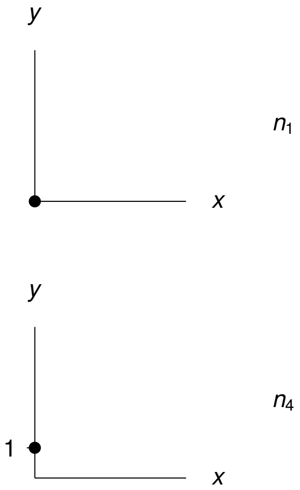

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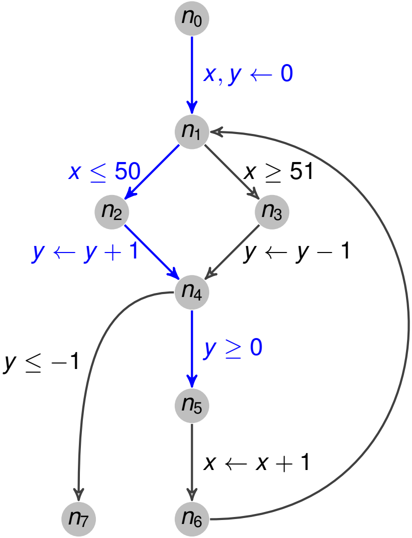

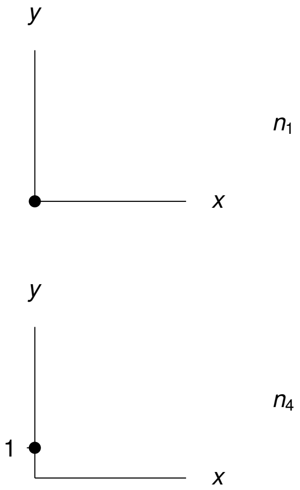

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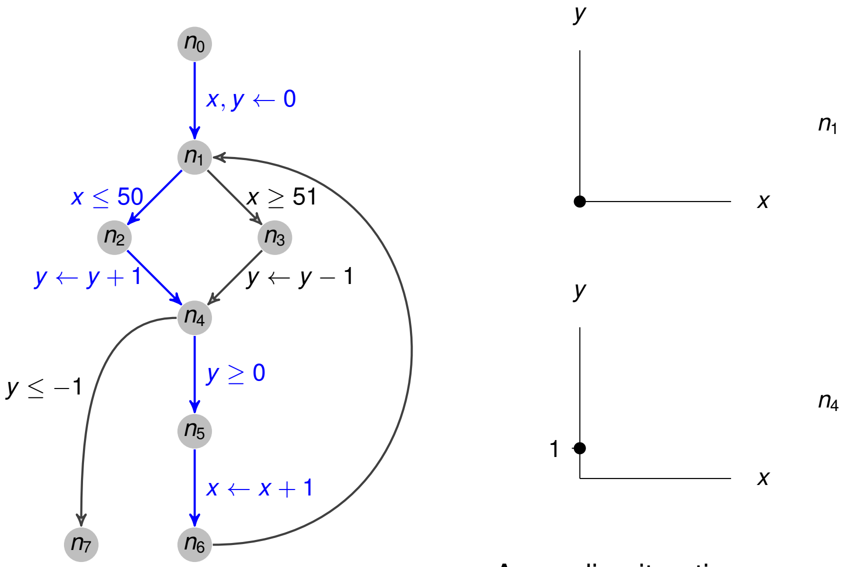

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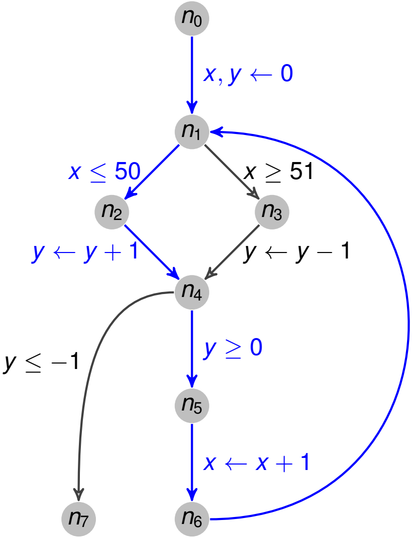

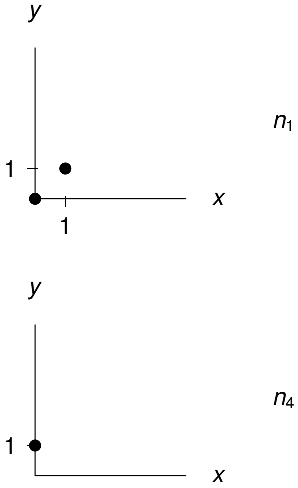

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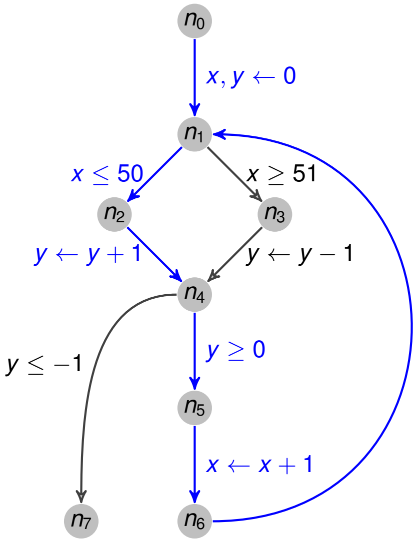

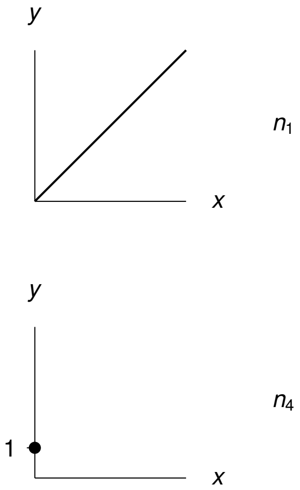

Ascending iterations

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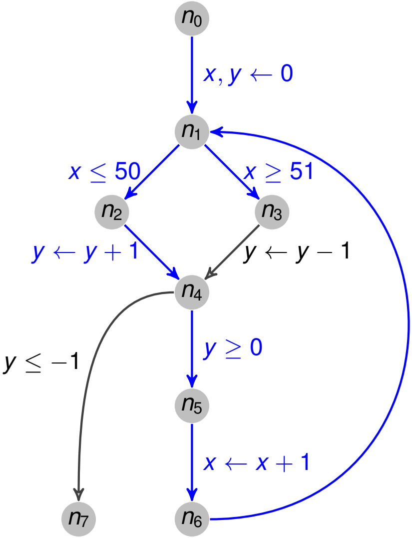

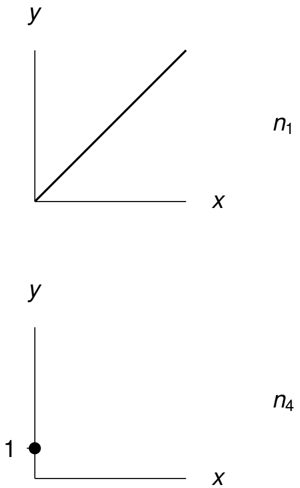

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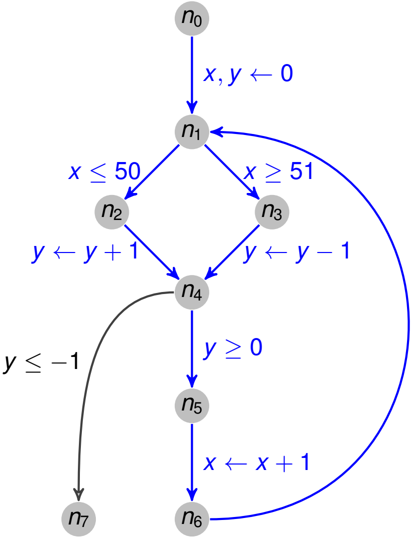

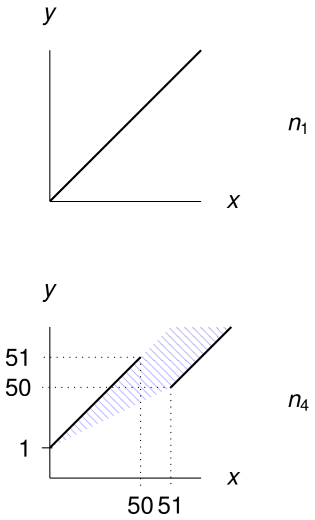

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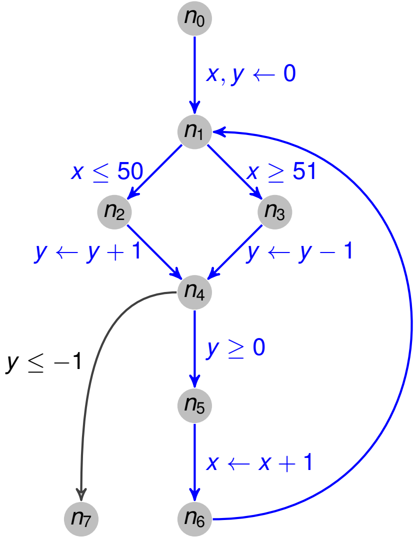

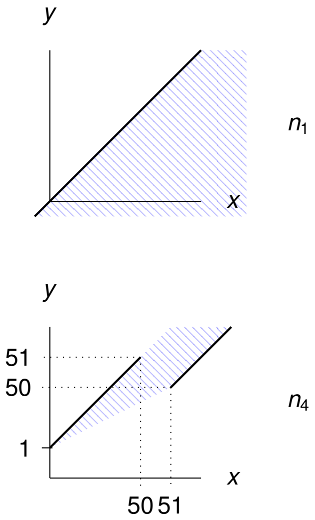

Ascending iterations

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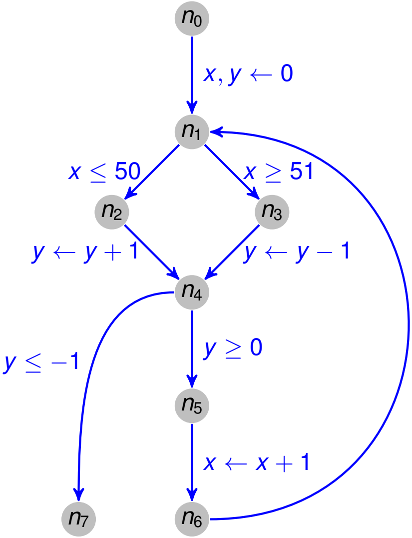

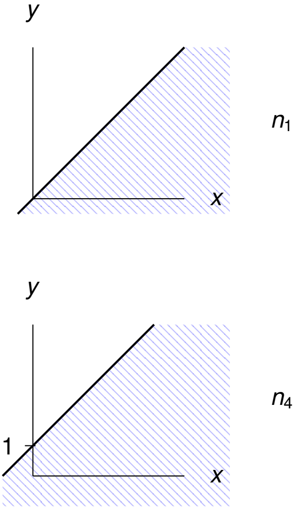

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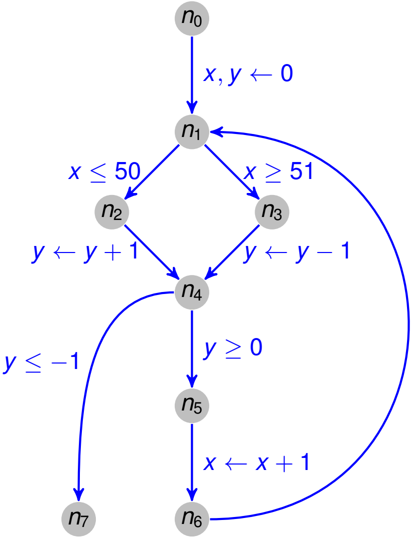

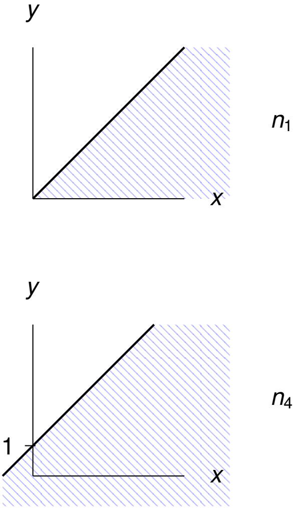

Descending iterations

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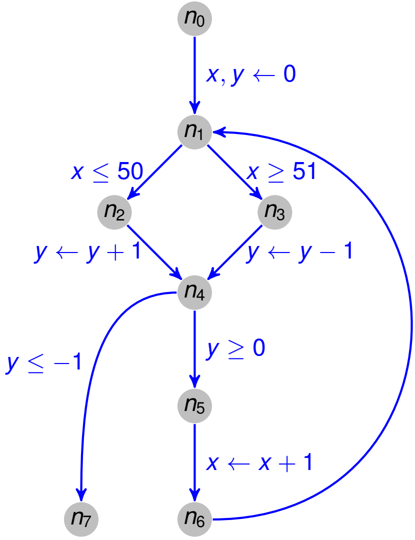
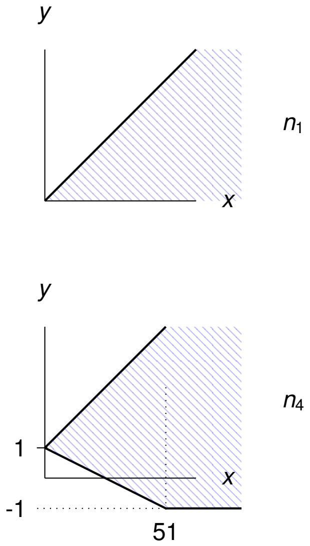




Descending iterations

Guided Static Analysis

D. Gopan & T. Reps, SAS'07

- separate loops into distinct phases.
- obtaining a solution for each loop phase before proceeding to the next.
- widening & narrowing at each loop phase.
  - Better precision

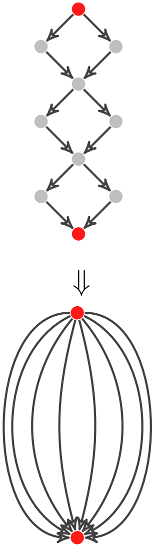
$\Rightarrow$ Ascending sequence of subsets of transitions

Summary

Principle of Path Focusing

D. Monniaux & L. Gonnord - SAS 2011

- Compute the fixpoint iterations on a multigraph
- Take a set P_R of nodes
- Distinguish all the paths between 2 nodes of P_R



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- Compute the fixpoint iterations on a multigraph
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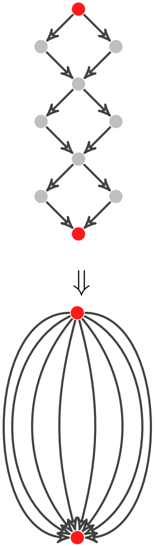
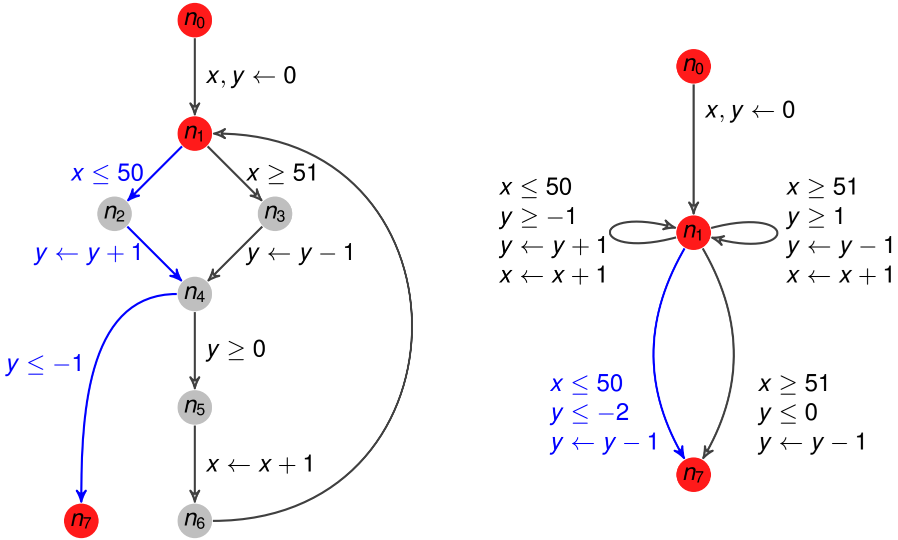
Exponential number of paths $\Rightarrow$

- We don't construct this graph explicitly
- We use SMT-solving to find interesting paths



Reducing the Graph

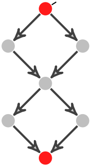


Using SMT-solving to Find New Paths

- SMT formula ρ expressing the semantics of the program
- ρ contains reachability predicates

"Does there exist a path starting in the invariant candidate, that arrives in a state outside the invariant ?"

$$f(p_i) = \rho \wedge b_i^s \wedge \bigwedge_{\substack{j \in P_R \\ j \neq i}} \neg b_j^s \wedge X_i \wedge \bigvee_{j \in Succ(i)} \left(b_j^d \wedge \neg X_j \right)$$



starting point

starting value

destination

destination value

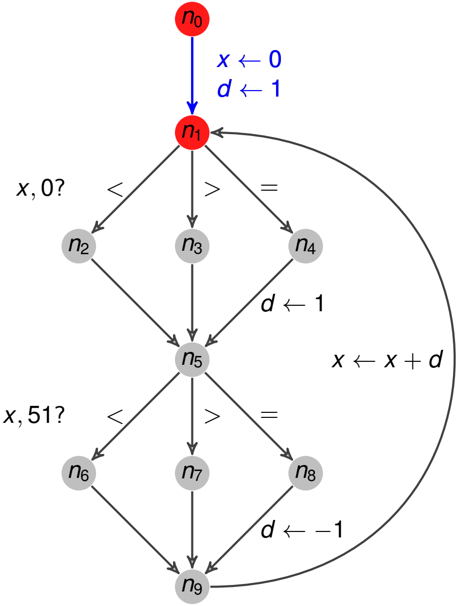
Example

```
int x = 0;
int d = 1;

while (true) {
        if (x == 0) d=1;
        if (x == 51) d=-1;
        x +=d;
}
```

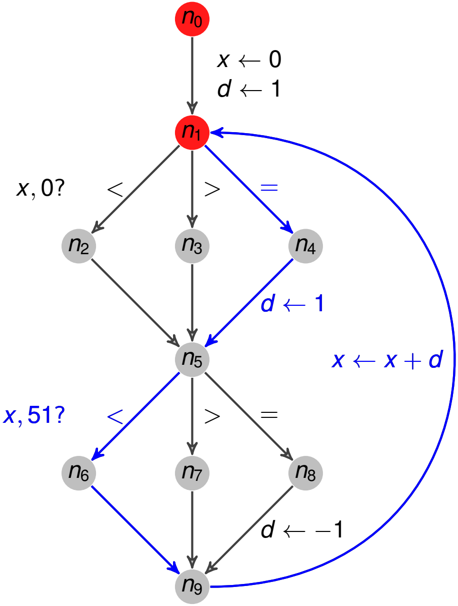
- *x* incremented until it is equal to 51,
- *x* decremented until it is equal to 0,
- restart...

Example


n_0
$x \leftarrow 0$
$d \leftarrow 1$
n_1
$x, 0?$
$<$
$>$
$=$
n_2
n_3
n_4
$d \leftarrow 1$
n_5
$x \leftarrow x + d$
$x, 51?$
$<$
$>$
$=$
n_6
n_7
n_8
$d \leftarrow -1$
n_9


d
1
n_1
x

Example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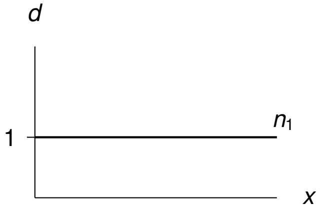
d
1
n_1
x

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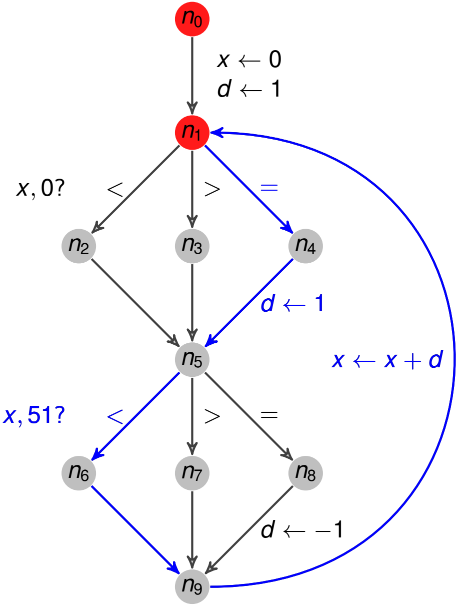

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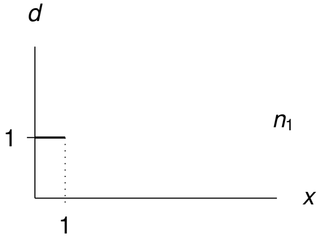
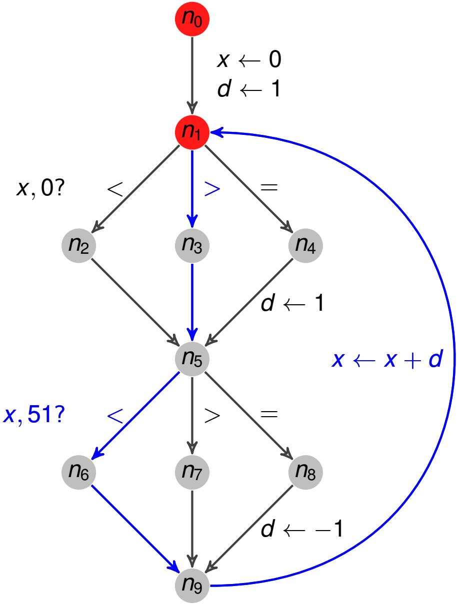


d
1
n_1
x
1

Example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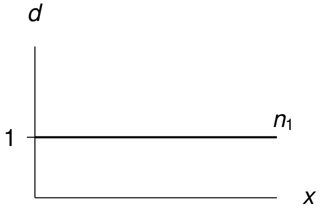
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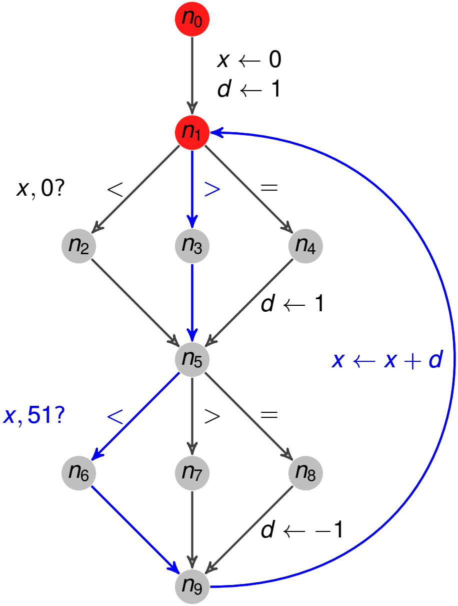
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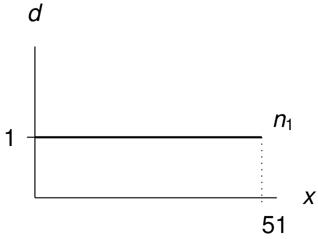
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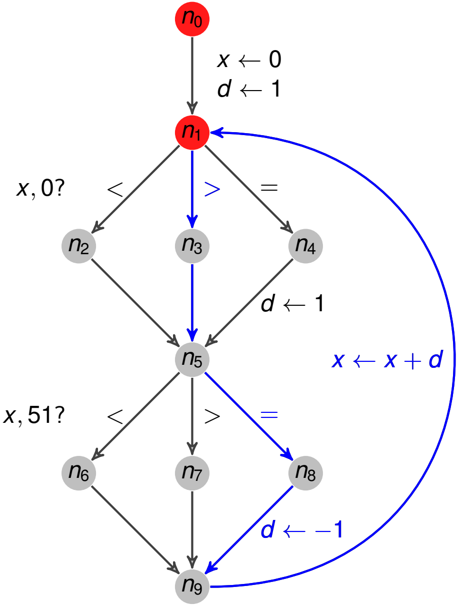
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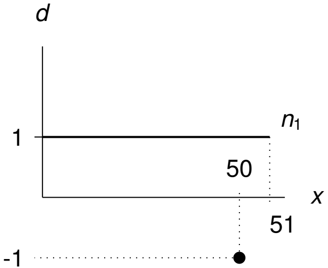
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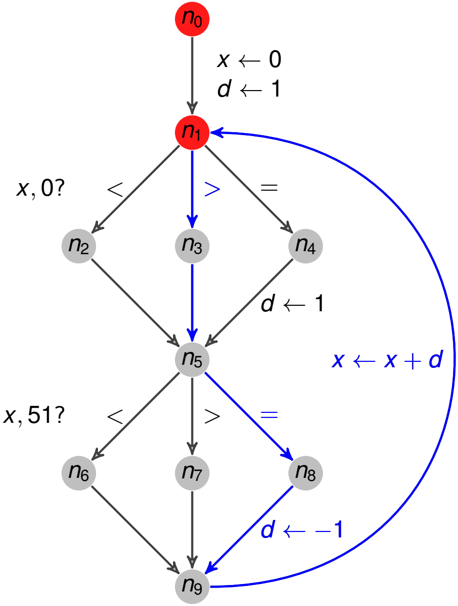
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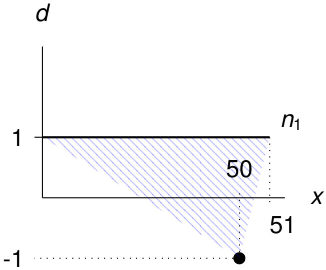
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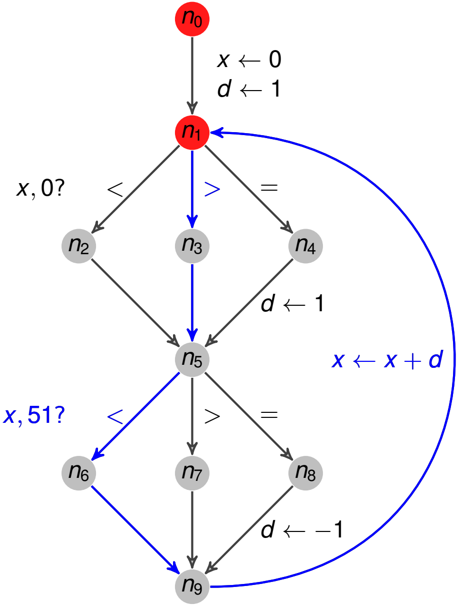

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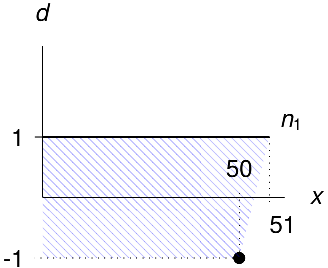


d
n_1
1
50
x
51
-1

Summary

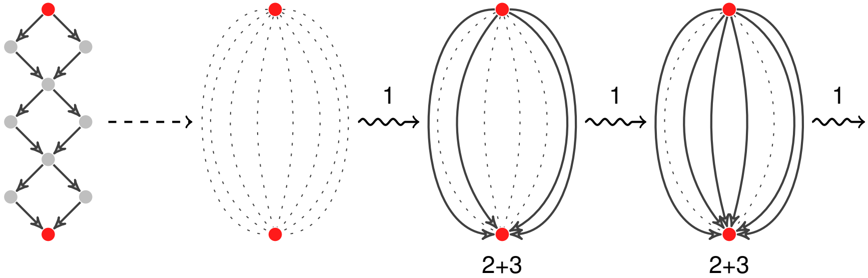
Our Contribution

We apply Guided Static Analysis over the reduced multigraph

Algorithm

3 phases:

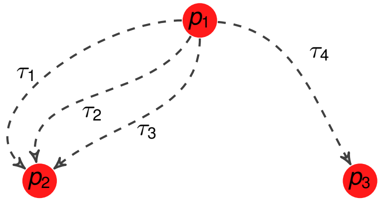
1. Compute new feasible paths
2. Path Focusing on the subset of the multigraph
3. Narrowing iterations



Computing New Paths

We store the set P of paths in a BDD.

New feasible paths starting at p_1:



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$p_3 : X_3$

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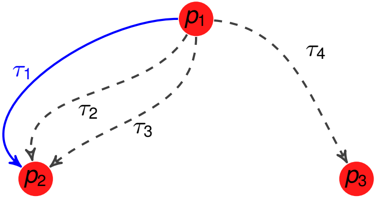

$p_2 : X_2 \sqcup \tau_1(X_1)$

$p_3 : X_3$

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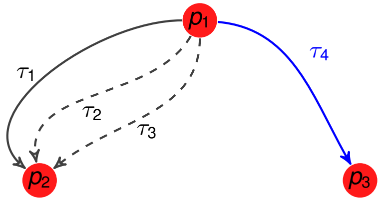

$p_2 : X_2 \sqcup \tau_1(X_1)$

$p_3 : X_3 \sqcup \tau_4(X_1)$

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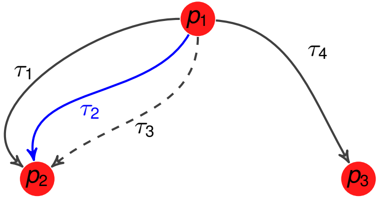

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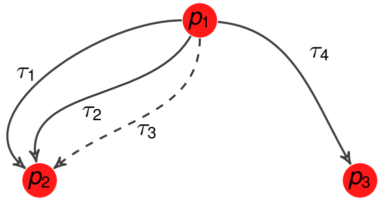
New feasible paths starting at p_1:



$p_2 : X_2 \sqcup \tau_1(X_1) \sqcup \tau_2(X_1)$

$p_3 : X_3 \sqcup \tau_4(X_1)$

τ_3 feasible, but:

$$\tau_3(X_1) \subset X_2 \sqcup \tau_1(X_1) \sqcup \tau_2(X_1)$$

Ascending Iterations

Path Focusing algorithm on the multigraph

But the formula is conjoined with P (subgraph):

$$f(p_i) \wedge P$$

Ascending Iterations

Path Focusing algorithm on the multigraph

But the formula is conjoined with P (subgraph):

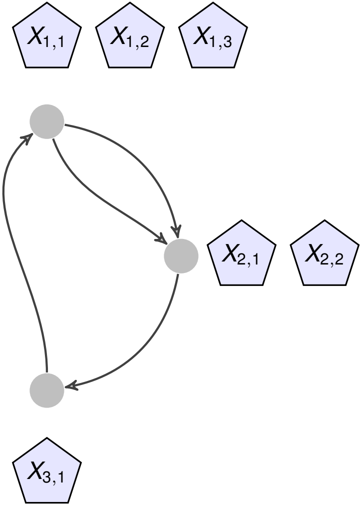
$$f(p_i) \wedge P$$

We also conjoin the formula with P for narrowing iterations...

Summary

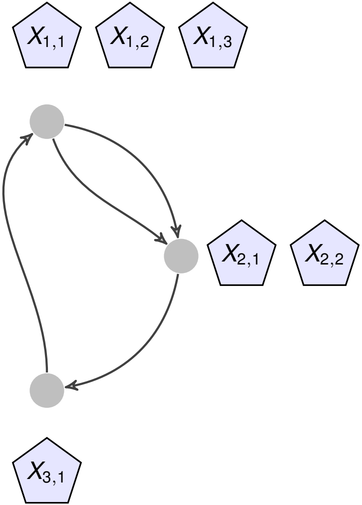
1. Introduction: Weakness of the standard approach & Guided Static Analysis
2. Using SMT-solving to focus new paths
3. Combining Both Techniques
4. Computing Disjunctive Invariants

Disjunctive Invariants



- How to choose the disjunct and the path to focus on?

Disjunctive Invariants



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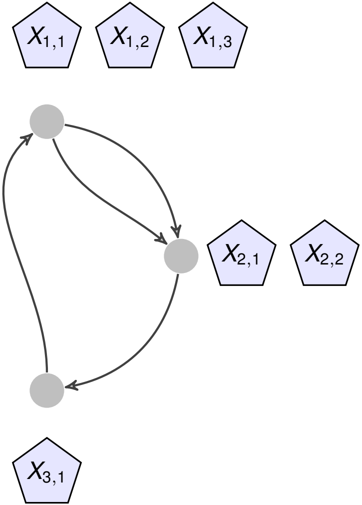
Disjunctive Invariants



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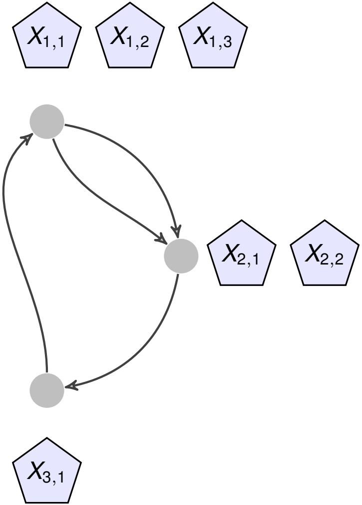
- How to choose which disjunct to join with?

Disjunctive Invariants



- How to choose the disjunct and the path to focus on?
  - **Use SMT**
- How to choose which disjunct to join with?
  - **Gulwani & Zuleger - PLDI 2010**

Using SMT to Focus Path and Disjunct

"Does there exist a path starting in one disjunct, that arrives in a state outside all disjuncts ?"

$$
\begin{aligned}
g(p_i) \quad = \quad & \rho \wedge b_i^s \wedge \bigwedge_{\substack{j \in P_R \\ j \neq i}} \neg b_j^s \\
& \wedge \bigvee_{1 \leq k \leq m_i} \left(d_k \wedge X_{i,k} \wedge \bigwedge_{l \neq k} \neg d_l \right) \\
& \wedge \bigvee_{j \in Succ(i)} \left(b_j^d \wedge \bigwedge_{1 \leq k \leq m_i} (\neg X_{j,k}) \right)
\end{aligned}
$$

One starting disjunct

Not in any destination disjunct

In the model, $d_k = true \implies$ we use $X_{i,k}$ as the starting disjunct.

Gulwani & Zuleger's Technique

Gulwani & Zuleger: The Reachability Bound Problem - PLDI'10

Disjunctive invariant for p_i: $\bigvee_{1 \leq j \leq m_i} X_{i,j}$

- $\delta_i \in [1, m_i]$
- mapping function $\sigma_i : [1, m_i] \times [1, n_i] \mapsto [1, m_i]$

$X_{i,\delta_i} \leftarrow$ initial states

The image of the j-th disjunct $X_{i,j}$ by the k-th path $\tau_{i,k}$ is joined with $X_{i,\sigma_i(j,k)}$.

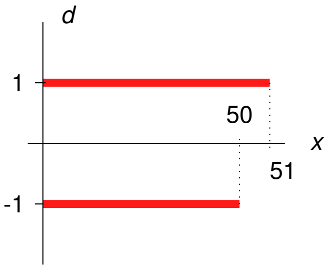
σ is computed dynamically (See Gulwani & Zuleger's paper)

Example

```
int x = 0;
int d = 1;

while (true) {
    if (x == 0) d=1;
    if (x == 51) d=-1;
    x +=d;
}
```

$$(d = 1 \wedge 0 \leq x \leq 51)$$
$$\vee(d = -1 \wedge 0 \leq x \leq 50)$$



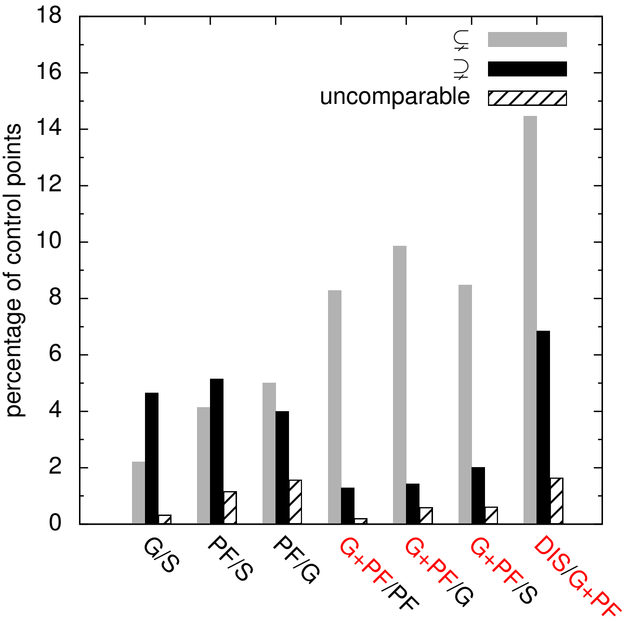
Experiments

These techniques are implemented in PAGAI: a prototype of static analyzer.

- LLVM IR as input
- Apron Library for the abstract domains
- SMT-lib 2 interface, Microsoft Z3

In TAPAS'12:
PAGAI: a Path Sensitive Static Analyser; Henry, Monniaux, Moy

Experiments on GNU programs and WCET benchmarks


percentage of control points
0
2
4
6
8
10
12
14
16
18
$\subsetneq$
$\supsetneq$
uncomparable
G/S
PF/S
PF/G
G+PF/PF
G+PF/G
G+PF/S
DIS/G+PF

Time

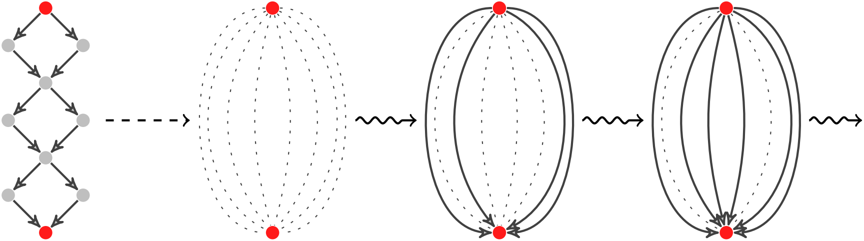
	Size		Execution time (seconds)				
Name	kLOC	$\|P_R\|$	**S**	**G**	**PF**	**G+PF**	**DIS**
a2ps-4.14	55	2012	23	74	34	115	162
gawk-4.0.0	59	902	15	46	12	40	50
gnuchess-6.0.0	38	1222	50	220	81	312	351
gnugo-3.8	83	2801	77	159	92	766	1493
grep-2.9	35	820	41	85	22	65	122
gzip-1.4	27	494	22	268	91	303	230
lapack-3.3.1	954	16422	294	3740	3773	8159	10351
make-3.82	34	993	67	108	53	109	257
tar-1.26	73	1712	37	218	115	253	396

Table: Execution times

Conclusion

- Path distinction avoids loss of precision due to join operators.
- Explicit exhaustive enumeration of paths can be avoided using SMT.
- This idea can be applied / combined with many existing techniques.

Questions ?



Dynamic Construction of σ

M: maximum number of disjunct
m_i: current number of disjunct

When $\sigma_i(j,k)$ is undefined:

1. if $\exists j', \tau_{i,k}(X_{i,j}) \sqcup X_{i',j'} = \tau_{i,k}(X_{i,j}) \cup X_{i',j'}$, we assign $\sigma_i(j,k)$ to j'
2. else:
   - if $m_i < M$, we increment m_i and define $\sigma_i(j,k) = m_i$
   - if $m_i = M$, we define $\sigma_i(j,k) = M$